Worked Answers: 30 Probability Questions

Question 1 — Probability

Define a probability and state its numerical bounds.

Hint: Think 0 through 1.

Working: A probability is a numerical measure of the chance of an event within a specified model; $0 \le P(A) \le 1$.

Answer: $0 \le P(A) \le 1$.

Avoid: Do not report 1.2 as a probability.

Question 2 — Random variable

What does a random variable assign to outcomes?

Hint: The output is numerical.

Working: A random variable maps outcomes of a random experiment to numerical values.

Answer: A numerical value for each outcome.

Avoid: The outcome and its probability are different.

Question 3 — Discrete vs continuous

Classify number of correct answers and an idealized measured time.

Hint: Counts vs continuous measurement.

Working: Correct answers take countable integers (discrete); idealized time is modelled on a continuum (continuous).

Answer: Count = discrete; time = continuous.

Avoid: Recording seconds to two decimals does not define the underlying idealized model.

Question 4 — Probability distribution

What does a distribution describe?

Hint: All possible events and how likely.

Working: It specifies probability for possible values/sets of a random variable under a model.

Answer: A complete probabilistic description.

Avoid: A histogram of observations is not automatically the theoretical distribution.

Question 5 — PMF

Explain $p(k)=P(X=k)$ for discrete X.

Hint: Probability at a count.

Working: A PMF gives exact probabilities at allowed discrete values; all masses are nonnegative and sum to one.

Answer: Exact-value probability mass.

Avoid: Do not apply a normal density height as PMF.

Question 6 — PDF

Can the height of a normal density at 70 equal P(X=70)?

Hint: Areas vs heights.

Working: No. For an ideal continuous normal X, P(X=70)=0; intervals are found from areas under the PDF.

Answer: P(X=70)=0.

Avoid: Density height is not an exact-point probability.

Question 7 — CDF

What does F(3) mean for a discrete count variable?

Hint: Includes all counts ≤3.

Working: F(3)=P(X≤3), the sum of probability masses at 0,1,2,3 when the support starts at zero.

Answer: P(X≤3).

Avoid: Do not exclude three.

Question 8 — Expected value

A fair two-outcome variable is 0 or 2, each with probability .5. Find E[X].

Hint: Multiply outcomes by probability.

Working: E[X]=0×0.5+2×0.5=1.

Answer: 1.

Avoid: An expectation need not be an outcome that occurs.

Question 9 — Complement

If P(A)=0.37, what is P(not A)?

Hint: Subtract from one.

Working: P(A complement)=1−0.37=0.63.

Answer: 0.63.

Avoid: A and its complement must sum to one.

Question 10 — Model selection

Choose normal/binomial/Poisson for ten independent pass-fail tests with common p.

Hint: Fixed number of Bernoulli trials.

Working: Binomial distribution with n=10 and the stated probability p, if independence and constant p are defensible.

Answer: Binomial.

Avoid: A count of successes is not always binomial if assumptions fail.

Question 11 — Z-score

For μ=70, σ=10, calculate z when x=85.

Hint: Subtract mean and divide SD.

Working: z=(85−70)/10=1.5.

Answer: 1.5.

Avoid: Use SD 10, not variance 100.

Question 12 — Normal left tail

For $X \sim N(70, 10^2)$, find $P(X < 80)$.

Hint: $z = 1$.

Working: $P(X < 80) = \Phi((80 - 70)/10) = \Phi(1) = 0.84134475$.

Answer: 0.8413

Avoid: A normal CDF returns area, not height.

Question 13 — Normal right tail

For $X \sim N(70, 10^2)$, find $P(X > 85)$.

Hint: $z = 1.5$; use upper tail.

Working: $P(X > 85) = 1 - \Phi(1.5) = 0.06680720$.

Answer: 0.0668

Avoid: Do not report left-tail 0.9332.

Question 14 — Normal interval

For the same normal X, find $P(60 < X < 80)$.

Hint: Use $\Phi(1) - \Phi(-1)$.

Working: $P = \Phi(1) - \Phi(-1) = 0.682689492$.

Answer: 0.6827

Avoid: Do not add the two cumulative probabilities.

Question 15 — Combinations

Compute $C(10,3)$.

Hint: $10!/[3!7!]$.

Working: $C(10,3) = 10!/(3!7!) = 120$.

Answer: 120.

Avoid: The ordered count is different from combinations.

Question 16 — Binomial exact

For $Y \sim \text{Binomial}(10, .30)$ find $P(Y = 3)$.

Hint: Choose 3 success slots.

Working: $C(10,3) \times .3^3 \times .7\blacksquare = 0.266827932$.

Answer: 0.266828

Avoid: Use mass (FALSE in spreadsheets), not cumulative.

Question 17 — Binomial cumulative

For $Y \sim \text{Binomial}(10, .3)$, find $P(Y \leq 2)$.

Hint: Sum k=0,1,2.

Working: Sum of binomial PMFs k=0 through2 = 0.382782786.

Answer: 0.382783

Avoid: At most two includes zero.

Question 18 — Binomial mean and variance

For n=10,p=.3 find E[Y], Var(Y), SD(Y).

Hint: Use np and np(1−p).

Working: Mean=10×.3=3; Var=10×.3×.7=2.1; SD=sqrt(2.1)=1.44914.

Answer: 3; 2.1; 1.4491.

Avoid: Expected three is not guaranteed three.

Question 19 — Poisson exact

For λ=2.5, calculate P(Z=3).

Hint: e^(-λ)λ^k/k!.

Working: P(Z=3)=e^-2.5×2.5³/3!=0.2137630172.

Answer: 0.213763

Avoid: Factorial k!, not λ!.

Question 20 — Poisson complement

For λ=2.5, find P(Z≥1).

Hint: 1−P(0).

Working: 1−e^-2.5=0.917915001.

Answer: 0.917915

Avoid: Do not confuse at least with exactly one.

Question 21 — Normal shaded area

What does a shaded normal area from 60 to 80 represent with μ70, σ10?

Hint: Interval probability.

Working: It represents P(60<X<80)=0.682689492, not an exact-point density value.

Answer: 68.27% approximately.

Avoid: Normal density height ≠ interval probability.

Question 22 — Compare normal models

Two normal models both mean70 but SD 5 versus 10. Which is more concentrated?

Hint: Smaller SD.

Working: The σ=5 model concentrates probability closer to 70; both have mean 70 but variances 25 and 100.

Answer: The model with SD 5.

Avoid: Same mean does not imply same spread.

Question 23 — Binomial assumptions

Would ten responses with success probability increasing after feedback necessarily follow Binomial(10,p)?

Hint: Check constant p.

Working: No: changing success probabilities and possible dependence violate the identical independent Bernoulli assumptions.

Answer: Not necessarily binomial.

Avoid: Counts alone do not justify binomial use.

Question 24 — Poisson assumptions

Support requests spike every evening. Is one constant hourly λ always appropriate?

Hint: Time-varying rate.

Working: No; a constant-rate homogeneous Poisson model may be inappropriate; inspect hourly patterns and other count models.

Answer: Check a time-varying rate model.

Avoid: Every count is not automatically Poisson.

Question 25 — Rate conversion

For 2.5 requests/hour, find expected count and P(0) in 30 minutes.

Hint: Multiply 2.5 by .5.

Working: λ30min=1.25; P(0)=e^-1.25=0.28650480.

Answer: λ=1.25; P(0)=0.286505

Avoid: Convert exposure before using the PMF.

Question 26 — Poisson approximation

For Bin(100,.03), compare P(0) with Poi(3) P(0).

Hint: Use λ=np=3.

Working: Exact binomial P(0)=0.04755251; Poisson approximation=0.04978707; absolute error=0.00223456.

Answer: 0.047553 vs 0.049787

Avoid: Approximation is not equality.

Question 27 — Continuity correction

Approximate P(Bin(50,.4)≤20) by normal continuity correction.

Hint: μ20, σsqrt12, boundary20.5.

Working: Exact binomial=0.56103493; corrected normal Φ((20.5–20)/sqrt12)=0.55738304; error=0.00365189.

Answer: Exact 0.561035, approximate 0.557383

Avoid: Do not use 20 instead of 20.5 when applying this correction.

Question 28 — Simulation vs model

If 1000 Bin(10,.3) experiments yield a frequency of exactly 3 different from .2668, is the model contradicted?

Hint: Sampling variation.

Working: No; finite independent simulations fluctuate. Compare observed fraction with theoretical PMF and consider sampling variation.

Answer: Differences are expected.

Avoid: Do not force simulated proportions to equal theoretical exactly.

Question 29 — Spreadsheet flags

How should NORM.DIST(80,70,10,FALSE) be described?

Hint: FALSE gives density for normal.

Working: It returns normal PDF height, not P(X=80). For P(X<80) use TRUE. For binomial and Poisson FALSE gives exact PMF.

Answer: Normal FALSE = density.

Avoid: Mislabeling continuous density as exact-point probability.

Question 30 — Integrated three models

Calculate P(60<X<80), P(Y=3), P(Z=3) for the project three variables.

Hint: Three different model families.

Working: Normal=0.68268949; binomial=0.26682793; Poisson=0.21376302. These concern distinct variables under their model assumptions.

Answer: 0.682689; 0.266828; 0.213763

Avoid: Do not model the same random variable with all three distributions.